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## **Why using a general model in Solvency II is not a good idea: An explanation from a Bayesian point of view**

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### **Abstract**

The passing of Directive 2009/138/CE (Solvency II) has opened a new era in the European insurance market. According to this new regulatory environment, the volume of own resources will be determined depending on the risks that any insurer would be holding. So, nowadays, the model to estimate the amount of economic capital is one of the most important elements. The Directive establishes that the European entities can use a general model to perform these tasks. However, this situation is far from being optimal because the calibration of the general model has been made using figures that reflects and average behaviour. This paper shows that not all the companies operating in a specific market has the same risk profile. For this reason, it is unsatisfactory to use a general model for all of them. We use the PAM clustering method and afterwards some Bayesian tools to check the results previously obtained. Analysed data (public information belonging to Spanish insurance companies about balance sheets and income statements from 1998 to 2007) comes from the DGSFP (Spanish insurance regulator).

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### **Keywords:**

Solvency II, PAM, longitudinal multinomial model.

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# 1 Introduction

Insurance is a way of protecting against risk (Wils 1994). Risk exists when people are exposed to the possibility of a future loss due to the occurrence and/or extent of which they do not know with certainty. The essence of the insurance mechanism is the reduction of risk by pooling (Benston and Smith 1976). Through the operation of the law of large numbers, uncertainty decreases when many similar but independent risks are brought together. If risk can thus be sufficiently reduced, an insurer can successfully offer to take over individuals' risks against a premium covering the expected loss, administrative costs and the remaining risks. That, is, insurance is a kind of business in which companies receive certain amounts of money as a pay for assuming risks. As in any economic activity, its main target is to be profitable. Nevertheless, and due to the nature of this kind of companies, they also need to be able to comply with the duties they have assumed with their customers, called policyholders. These companies should have enough financial strength because they need to meet with all the possible contingencies associated with their activities. So, it is needed to analyze the solvency or ability to ensure this kind of companies will be able to face on time with their financial duties.

Although solvency and profitability could be seen as opposite characteristics, it is true that companies need the former if they want to reach the latter. In this context, supervisor authorities have always been searching for a set of rules and indicators that try to reflect the strength of the insurance undertakings.

The worry about this issue is not new in the European Union context. In fact, the first rules about this matter were passed during the 70s of the last century. Directives 73/239/EEC and 79/267/EEC for life and non-life insurances, respectively, obliged the companies to estimate the amount of capital they would need to face with sudden events. These rules were thought as minimum common requirements of capital for all Member States although each of them were free to lay down a more severe set of rules, if it were its desire. This regulation was replaced at the beginning of this century by a set of more strict Directives, which were called Solvency I. In the new regulatory environment, besides changes in the assessment of solvency risk margin for life and non-life insurers, there were established rules for activities such as reinsurance, control of insurance activities into financial conglomerates, or reorganizations and winding up of institutions. However, despite the effort to update legal rules to the context, capital level were still calculated according fixed rules that were applicable to any insurer, no matter the concentration and nature of risk it was holding.

Since the beginning of this century, European insurers are immersed in a process to change the way to estimate the level of their own funds. The aims of the process are (FSA 2006):

- to strength policyholder protection through capital requirements which can provide early warning of deterioration in solvency levels

- to provide insurance companies freedom to fix their own risk profile, as long as they hold commensurate risk capital
- to align economic and regulatory capital
- to stimulate further improvement in the quality of risk management.

This reform process did start in 2003, when the European Commission prepared an explanatory note about the design of the system to be used in the future for calculating the solvency capital<sup>1</sup>. It is intended the system will apply in the insurance area the principles that ruled the reform of Basel II in the banking field. Therefore, and as in such scheme, the system is built around three pillars. Other key aspects of Solvency II are that solvency margins structured around two main figures: one, that we could be considered as economic capital, which would be the amount associated with the risk-bearing. This is what is called the Solvency Capital Requirement, or SCR; the second one, that we could be considered as legal capital, which would be the minimum required amount. It is called the Minimum Capital Requirement or MCR.

The key point of the new system is the change of criterion for calculating the amount of the capital of solvency, because its role changes from calculating the solvency capital as a function of the risk of subscription -primes- to make it dependent on the level of risk supported in all and each one of the spheres in which the insurance activity takes turn. Despite the newness of the approach, de Haan and Kakes (2010) have shown that Dutch insurers set their capital levels considering risks instead of legal requirements now in force much before the new Directive begins to oblige. This process of change has concluded with the pass of Directive 2009/138/CE (Solvency II Directive). The whole scheme will be completed in the future with the design of a mechanism for measuring the solvency of the undertakings. This tool will be able to estimate the amount of own resources in each company according to the risks taken by them. In order to achieve this target, CEIOPS (Committee of European Insurance and Occupational Pensions Supervisors) has executed five empirical studies, called QIS (Quantitative Impact Studies). The use of these analytical tools provide a huge advantage: their easiness of use. Whatever the company or its risk policy were, it will be enough to apply the general model to assess its level of capital required. However, they have one big drawback: because the model is calibrated from data proceeding from the sector as a whole, it will adequately represent the average behaviour of the industry. So, if the risk policy set up a profile different than that of the industry average, then the general model will calculate an amount of capital that will have little or no connection with the situation of the company.

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<sup>1</sup>European Commission, Internal Market D.G. (2003): Solvency II - Reflections on the general outline of a framework directive and mandates for further technical work. MARKT/2509/03. Brussels, 3 March 2003.

It seems clear that if the analysis of insurance undertakings could lead us to deduce that there are different realities among them, then it could be obliged to accept that the use of a general model for assessing the company's solvency capital would not give the results desired by the new regulations. As a final conclusion, it would be much better the elaboration of an internal model that reflects the risk profile of each company or, at least, a general model for a group of entities with similar features.

This paper tries to study the possible heterogeneity that can exist into an specific insurance market. If it was so, it can not be appropriate to use a general model for the whole set of entities operating in that market. To do this, Spanish data have been used. They came from the Spanish Insurance regulator database for balance sheets and income statements since 1998 till 2007. The remainder of the paper is set out as follows. Section 2 is focused on the review of the literature about differences in results and behaviour due to factors such as lines of business or legal structure of companies. Section 3 describes the data used in the analysis. It is also explained the methodology we have followed to do the classification. Section 4 introduces two different models to predict the probability of belonging to a certain group, using a set of explanatory variables. Finally, Section 5 concludes with some remarks.

## 2 Background about classification

There are many ways to use the statistical analysis in order to get homogenous blocks of information into a heterogeneous set of data. McCarty and Hastak (2007) make a review about the most extended techniques in data mining for segmentation. They emphasizes in RFM (recency, frequency and monetary value), CHAIDS and logistic regression. This set of techniques is generally used in areas such as Marketing or financial analysis oriented to assess the degree of solvency in the companies that composes an specific economic sector. In the first field, it would be intended to classify the customers of a company according to their characteristics in order to estimate their degree of acceptance of certain product, new or into the portfolio. Following this trend, it can be found Wu, Kao, Su, and Wu (2005), Abrahams, Becker, Sabido, D'Souza, Makriyiannis, and Krasnodebski (2009) or Hsu (2010) most recently. In the first of them, KDD/DM (knowledge discovery in databases and data mining) is used to look for the rules that allow to identify potential customers for new or existing products of an insurance company. They used algorithms such as ID3 (Quinlan 1986). Abrahams, Becker, Sabido, D'Souza, Makriyiannis, and Krasnodebski (2009) focuses their attention in the use of algorithms based in decision trees, such as CART (Breiman, Friedman, Olshen, and Stone 1984), CHAID (Kass 1980) or ID3. Hsu (2010) uses completely different methodologies, such as grey clustering or back propagation network. Flórez-López and Ramón-Jerónimo (2009) are between Marketing and financial applications. They use market segmentation as a tool with which CPA (customer profitability accounting) can estimate the success of the CRM (customer relationship management) through the mixture of marketing feature selection, cus-

tomers segmentation through univariate and oblique decision trees and scenarios.

A huge variety of methodologies and papers can be found when this kind of techniques is used in financial areas. Anghelache and Armeanu (2008) and Dedu, Armeanu, and Enciu (2009) use simple models based in principal components and discriminant analysis, respectively, to describe the Romanian insurance market. In line with the main goal of this paper, it can be found several papers in which different statistical methodologies are used to assess the solvency of an insurance undertaking. Trieschmann and Pinches (1973) study a sample of property-liability insurance companies in Missouri with discriminant analysis. Kramer (1996) uses OLR (ordinal logistic regression) to forecast financial solidity of Dutch non-life insurers, classifying them in three categories (strong, moderate and weak). BarNiv and McDonald (1992) is a survey about the most common techniques employed to classify companies according to their financial strength: It also includes a new methodology based in exponential generalized distribution of second kind (EGD2). Rating is a matter close to solvency. This tool tries to classify companies according to their insolvency risk. Van Gestel, Martens, Baesens, Feremans, Huysmans, and Vanthienen (2007) made an internal model based on OLR and SVM (support vector machine) to assess the rating that insurers would have. They analyzed the companies that previously were rated by Standard & Poor's. If the attention is focused in the legal nature of the companies, the industry offers an environment within which the companies usually adopt one of two major types of ownership structure -the stock and/or the mutual form. Shareholders own stock companies, whereas mutual companies have no equity capital and are nominally owned by their customers, the policyholders (Cummins and Weiss 1991).

As in Marketing or in financial analysis for solvency, it is also possible to find papers showing different behaviors depending on the legal nature of the companies. Brockett, Cooper, Golden, Rousseau, and Wang (2004) use DEA (Data Envelopment Analysis) coupled with rank order statistics to study the relative efficiency in property-liability companies. That is, they try to show if both types of companies give the same importance to matters such as efficiency or solvency.

### **3 Data and methodology in Clustering**

The purpose of this study will be to examine empirically the solvency risk of Spanish insurance companies from the assets and liabilities, reported in their balance sheets and how this kind of risk might depend of their organizational form, size or business line. Data were extracted from public information supplied by the Spanish insurance regulator (DGSFP according to its Spanish acronym). It includes all the insurance companies that have being operating in Spain since 1998 till 2007, in all lines of business.

It is true that for the Solvency II calculations the whole balance must be assessed at market prices, and also profits and losses. But to do it in this way means that it

would be needed not only to know the kind of investment each company has made and who are its policyholders but also aspects such as duration and profitability of their investments, or the probability distribution of claims, associated costs and the expected time for settlement among others. Definitively, it would be necessary project towards the future the predictable cash flows, and proceed to update them later. However, public information available does not provide such data but only accounting-P&L issues, technical and nontechnical accounts, solvency margin and coverage of technical reserves. Therefore, we have proceeded to analyze the behaviour of firms and their risk position based on ratios. As noted above, for the analysis we used the public information supplied by the DGSFP for the years between 1998 and 2007. The selection of this interval is due to a change in the accounting standards for insurance companies beginning in 2008. The total number of companies operating between these two years is 456. However, the study has been done with the 223 insurers with information in all the years and for the whole set of selected variables. These companies represent on average 86.0% of total assets, so we can consider this sample as representative of the whole Spanish insurance sector. Table 1 shows the number of companies classified according to their legal form.

Table 1: Companies classified according their legal form

Legal form	companies	%
Joint Stock Companies	156	70.0
Social Welfare entities	32	14.3
Mutual companies	32	14.3
Reinsurers	2	0.9
Foreign branches	1	0.4

*Source:* DGSFP and own elaboration

The ratios that were used are related to the assets, liabilities and income. Their names and composition are shown in Table 2:

Some of these ratios are general to any company-for example, R2 is ROA or R3 is ROE-, while others are specific to the insurance business, such as R9 -combined ratio-. It has to be said that all ratios referred to returns have been calculated as a flow divided by a balance. In these cases, the denominator is the average between the balance of a variable on a year and the one in the previous year. For this reason, the real time interval for analysis has begun in 1999 and has finished in 2007.

We take the computed ratios  $R1$ ,  $R2$ ,  $R3$ ,  $R4$ ,  $R5$ ,  $R6$ ,  $R7$ ,  $R8$ ,  $R9$  and  $R10$ , and four categorical variables: one that identifies each company in terms of its type (from 1 to 5, that is, 1 for stock companies, 2 for social welfare entities, 3 for mutual companies, 4 for reinsurers and 5 for foreign branches, and three binary variables which identify whether the company activities are *life insurance*, *non-life* or both kind of business. All variables have been measured during nine years (from 1999 to 2007).

Table 2: Definition of ratios used in the analysis

Ratio	Numerator	Denominator
R1	Return of financial assests	Total financial assets
R2	Total returns before taxes	Total assets
R3	Total returns after taxes	Total equity
R4	Total investments	Total assets
R5	Own capital and retained earnings	Total liabilities
R6	Total assets	Total debt
R7	Reserves	Gross Premiums
R8	Reserves	Total Gross Indemnities (TGI)
R9	TGI + Gross expenses	Gross Premiums
R10	Gross Premiums	Total assets

*Source:* own elaboration

As a first stage, we may consider a  $k$ -means clustering method to find out which are the possible groups in which we can classify the different behaviors of the companies depending on their legal nature. A good choice is to use the methodology of longitudinal  $k$ -mean clustering, as in Genolini and Falissard (2010). However, the potential presence of outliers makes preferable the use of a more robust version of the  $k$ -means clustering method such as *Partitioning Around Medoids* (PAM) (see Kaufman and Rousseeuw 1990) by adapting this methodology to the longitudinal case.

Therefore, we derive for each company a vector consisting of the original variables measured along the nine years under consideration, resulting in a total of 126 qualitative and quantitative variables. We use the PAM methodology by computing the Gower distances (Gower 1971) among all the companies. We run the PAM method for different number of clusters and we selected the optimum number in terms of the highest value of the Calinski-Harabasz pseudo F-statistic (see Calinski and Harabasz 1974). The local maxima are obtained with 3 and 5 groups.

In the three-clusters case, the differences amongst them is only due to the line of business. One of them joins all the non-life insurers, the second one all the life insurers and the remaining group includes these companies with activities in all kind of insurances. In this case, neither the legal form of companies nor the size do have any effect in the classification. However, in the five-clusters case, the size of companies do have a considerable impact in how companies are grouped together. So, the former groups of all life and non-life insurers are now divided into two different clusters each. The new division can be related to the size (big vs small companies). The cluster that includes companies with both lines of business remains as before. According to the centroids in groups, the technical and financial characteristics of big companies are similar in both life and non-life insurers. That is, comparing with the small companies, all of them show:

- higher financial returns (ratio R1)
- higher relative volumen of technical reserves (ratios R7 and R8)
- higher combined ratio (ratio R9)
- lower return to equity (ratio R3)

According to their most representative features, the five groups, or clusters, can be identified as follows:

- Class 1: non-life insurers, small size. This class has been taken as the reference one in the model
- Class 2: non-life insurers, big size
- Class 3: with all kind of line of business insurers
- Class 4: life insurers, big size
- Class 5: life insurers, small size

The value of the centroids for each group and year can be found in Annex I.

## 4 Models to evaluate the classification

Once the groups have been obtained, the next step consists of studying the relationship amongst those groups and the explanatory variables considered in the database. Most of the statistical methods applied in longitudinal categorical data are related with binary or Poisson models. They generally use marginal models, which consider the correlation between repeated measurements (for a general revision, see Ashby, Neuhaus, Hauck, Bacchetti, Heilbron, Jewell, Segal, and Fusaro 1992). Marginal models can be considered as an approximation to generalized linear mixed models (Breslow and Clayton 1993); these models are more adequate when the main interest of the study is focused on the individuals. Estimation for the generalized linear mixed model can be undertaken in several different ways. From a classical point view, several iterative methods and *Expectation Maximization* (EM) algorithms are available (see McCulloch 1997), and from a Bayesian point of view, Bayesian hierarchical models can be applied. As Natarajan and Kass (2000) point out, this last approach has several advantages over the classic methodology. Among them, Bayesian hierarchical models obtain restricted maximum likelihood estimates and they take account of the uncertainty in estimating the variance components.

In this section, we apply a similar approach to that of Pettitt, Tran, Haynes, and Hay (2006) who worked in the context of a model for immigration characteristics. Here, we consider the relationship between the class of company that we derived in section 3 and the categorical and continuous explanatory variables included in our study, by



specifying a Bayesian hierarchical model for the multinomial response. The classes are those related to clusters obtained in the classification.

We have used the library **MCMCglmm** (see Hadfield (2010)) for the analysis of unordered multinomial data. Originally, this library was made to solve problems found in Quantitative Genetics applications. Nevertheless, the nature of the problem analysed in this paper allows its use. So, it is possible to create a program for this case and translate it to the standard requirements of the library.

The model is parameterized as a log-odds ratio against a baseline category. We assume that, for  $i = 1, \dots, n$  companies,  $t = 1, \dots, T$  years and  $j = 1, \dots, J$  categories

$$\sum_{j=1}^J P(y_{it} = j) = 1. \quad \text{for } t = 1, \dots, T$$

When there are more than two categories we need to assume a *baseline* category and the remain  $J - 1$  as *latent* ones. So the log-odds ratio of the base category versus the rest of categories is defined as,

$$l_{i,j,t} = \log \left( \frac{P(y_{i,t} = j)}{P(y_{i,t} = \text{base})} \right)$$

for  $i = 1, \dots, n$  observations and  $k \in J - 1$  categories. Then,

$$l_{i,j,t} = X_{it}^T \beta_j + \alpha_{i,j}$$

$X_{it}^T$  is a matrix with all the explanatory variables considered in our model, that is, those that identify the ratios and the classes. When we consider the class  $j$  of company, the regression effects are constant but each company  $i$  is considered as a group of responses over the observed years ( $t = 1, \dots, T$ ). Term  $\alpha_{ij}$  is included and it can vary between companies. Given the class  $j$ , the term reflects the observed variability that is not constant over time. In this way, the observations of the each company, through several periods of time, are considered as replicates and it is introduced in the model as a random effect.

We consider diffuse prior distributions over the parameters; this choice is specified in the prior information in the **MCMCglmm** package by defining a multivariate error structure. In multinomial models with a single count it cannot be estimated from data so it must be fixed. Following recommendations in Hadfield (2011), we fix the residual variances to 1 and the covariances to 0, and the same structure in the random effects part. Several models have been considered. The difference among them is the number of variables to be included in each. The final model has been chosen according the value of DIC. The results of the estimation according to this criterion can be seen in table 3.

The variables with the most relevant effects are R5 and R10. In other cases, the highest posterior density (HPD) intervals do not include zero values also, such as those referred to Class 3, R3, R6 and R8. These results can suggest a poor fitted model

Table 3: Multinomial model: results of the estimation

	Mean	SD	95% lower HPD	95% upper HPD
<b>Class.2</b>	2.4452	1.7518	-1.0524	5.2604
<b>Class.3</b>	2.7114	1.3614	0.4261	5.2209
<b>Class.4</b>	0.6993	1.8234	-2.4509	3.4720
<b>Class.5</b>	1.0684	1.4315	-1.3820	3.3240
<b>R1</b>	0.3979	0.2756	-0.0937	0.8125
<b>R2</b>	2.5022	2.4236	-1.7629	7.3957
<b>R3</b>	0.4208	0.1606	0.1119	0.7089
<b>R4</b>	-0.8346	2.0206	-3.8299	2.1339
<b>R5</b>	-15.2759	2.1112	-17.8802	-11.0009
<b>R6</b>	0.2688	0.1227	0.0206	0.4493
<b>R7</b>	0.0761	0.0912	-0.0776	0.2532
<b>R8</b>	0.4501	0.0856	0.2719	0.5773
<b>R9</b>	0.1674	0.4982	-0.8313	0.9717
<b>R10</b>	-2.0260	0.2826	-2.5813	-1.5293

Source: own elaboration

because very few relevant variables are relative to insurance activity. In fact, we can just point out the coefficients linked to R5 (turnover of premiums) and R8 (Reserves to TGI). The remaining non-zero variables are referred to characteristics that can be used in the analysis of any kind of company. Besides, the only class that can be rightly identified with this model is the third one. These two reasons recommend to search for another kind of methodology to fit the model.

Now we are going to consider a standard multinomial model as a base model, which follows the approach and notations of Ntzoufras (2009). As we are dealing with time series, it can be adequate to consider the existence of some kind of autocorrelation in data. For this reason, we introduce an autoregressive dependence scheme among the parameters to model the longitudinal nature of data; this modification can be included inside the techniques of dynamic general linear models (see West and Harrison (1997)).

We assume that the response variable at time  $t$ ,  $\mathbf{y}_{ti} = (y_{ti1}, \dots, y_{tiJ})$  has  $J$  levels, where  $y_{tij}$  denotes the frequency of the  $j$ -th level at time  $t$ , then the dynamic multinomial logistic model can be written as

$$y_{tij} \sim \text{Multinom}(J, \pi_{tij}) \text{ where } \sum_{j=1}^J \pi_{tij} = 1,$$

or alternately,

$$\mathbf{y}_{ti} \sim \text{Multinom}(\pi_{ti}, 1).$$

Then, for  $j = 2, \dots, J$  levels, and  $k = 1, \dots, m$  explicative variables,

$$\log \frac{\pi_{tij}}{\pi_{ti1}} = \eta_{tij} = \beta_{t0j} + \sum_{k=1}^m \beta_{tkj} \gamma_{tkj} x_{tik} \quad (1)$$

where  $\pi_{ti} = (\pi_{ti1}, \pi_{ti2}, \dots, \pi_{tiJ})'$  is the vector of the probabilities for each level of variable  $\mathbf{y}_t$  for individual  $i$  with  $\pi_{ti1} = 1 - \sum_{j=2}^J \pi_{tij}$  and  $\gamma_{tkj}$  are binary indicators which identify the structure of the model and which variables specify or affect each odds (see, Agresti 2002).

If we express (1) in terms of response probabilities, then

$$\pi_{tij} = \frac{e^{\eta_{tij}}}{\sum_{j=1}^J e^{\eta_{tij}}}$$

with  $\eta_{ti1} = 0$  for  $i = 1, 2, \dots, n$ .

The restriction  $\eta_{ti1} = 0$  can be indirectly imposed by setting all coefficients of the first linear predictor  $\beta_{ti1}$  equal to zero (see Ntzoufras 2009). With regard to the autoregressive relation between parameters, it can be modeled as:

$$\beta_{tkj} = \beta_{(t-1)kj} + e_k$$

where  $t = 2000, \dots, 2007$

$$e_k \sim N(0, \sigma_0)$$

We have programmed the model using **Jags** (see Plummer (2003)). One advantage of using **Jags** is that it constructs the full conditional distributions and carries out the Gibbs sampling from the model specifications as **WinBugs** does. Another additional benefit of this program is that it can be run in more OS than Windows. The code is available from the authors upon request. The results of the estimation for each group can be seen in Annex II.

The main difference with the former model is that each group has its own expression and the parameters referred to each ratio can be different amongst them. Highest Posterior Density (HPD) intervals (with 0.95 probability) do not include zero values at least for one group (i.e., R1 and R2 for group 3 whereas R3 in group 2). In the opposite case, the intercept, R5, R8, R9 and R10 ratios do not include zero values in HPD intervals in all groups. That is, in all cases, variables such as the capital structure, the proportion between Reserves and indemnities, the combined ratio or the premium turnover are useful to distinguish amongst companies. If we analyze the results of the estimation in each group, they reflect the differences among each class and the one used as reference. We can conclude that:

- class 2 companies (big non life insurers) show a similar behaviour in financial returns, ROA and the relationship between Reserves and gross premiums. So, the effect of size is reflected in the structure of both sides of their balancesheets and in technical aspects such as the combined ratio or the premium turnover

- class 3 show similar ROE and leverage ratio that these in the reference group
- similarities in class 4 are focused on performance ratios and on the weight of investments into the left side of the balance.
- finally, the similarities between insurers into class 5 and those in the reference group are related to performance and leverage ratios.

## 5 Concluding remarks

The pretension of using a general model that would be used for the whole sector is based on the idea that the calibration of this tool may rightly reflect the behaviour of a company. This assumption would be true if and only if this hypothetical insurer had a set of results quite similar to those in the average of the sector. However, the empirical evidence seems to show that this assessment is far to be true. Even though the general model in QIS studies is structured in independent risks and that each company should estimate its own resources according to its risk portfolio, that model does not consider some intrinsic aspects such as the size of the company.

From a methodological point of view, we have first considered a robust cluster model (PAM) by using Gower's distances. This approach seems quite adequate because it not considered an a priori number of groups, allows the use of general dissimilarities matrices and, finally, it is computationally very efficient. Once the groups have been obtained and a possible interpretation for their composition were found, the study of the relationships amongst ratios has been the next step. The variables with the greater explicative power in every relation change depending on the groups of insurers we were considering. The groups are those formed in the former stage. Firstly, we have used a multinomial longitudinal model. This kind of models appears in problems linked to biological questions, such as Population Genetics (see e.g. Sorensen and Gianola (2002)). In this context, Bayesian methodology has proven to be quite useful. Translating this approach to the problem treated in this paper, insurance undertakings can be considered as replicas with no time relationships amongst measures. As an alternative methodology, we have considered a dynamic multinomial logit model. It was applied to the analysed companies during the whole time interval considered in this study. This kind of model can be considered as a dynamic linear model (DLM). It is widely used in longitudinal questions because it is allowed to accept that time dependence amongst observations can exists. Besides, it is possible to identify the relevant variables (ratios in our case) involved in the proposed model. Our study seems to show that there are two variables with a considerable incidence when classifying companies amongst different groups. The results suggest that the line of business and the size are the two main variables we should consider if we like to build a model that assess the level of capital depending on the holding risks. So, the final conclusion is that it seems to be inadequate to estimate the economic capital of an specific insurer in a certain country using a model calibrated with data of the whole European insurance sector.

## Annex I: centroids of clusters

Class 1: non life insurers with small size

Variable	1999	2000	2001	2002	2003	2004	2005	2006	2007
<b>R1</b>	9.7%	7.3%	17.8%	-5.7%	-52.4%	2.0%	2.6%	7.7%	3.7%
<b>R2</b>	12.7%	5.7%	6.2%	6.3%	7.7%	9.4%	10.2%	10.9%	8.3%
<b>R3</b>	14.5%	6.5%	6.8%	6.7%	9.0%	12.4%	12.5%	13.5%	9.3%
<b>R5</b>	7.6%	6.5%	6.3%	6.9%	7.6%	7.9%	8.3%	8.0%	6.5%
<b>R4</b>	65.7%	67.1%	68.9%	67.5%	68.8%	68.4%	70.0%	69.1%	72.4%
<b>R5</b>	63.9%	63.4%	64.0%	62.3%	61.7%	61.1%	62.1%	62.8%	64.0%
<b>R6</b>	382.3%	375.5%	386.1%	351.1%	328.7%	327.5%	337.1%	339.3%	381.4%
<b>R7</b>	32.4%	31.9%	30.4%	29.9%	30.8%	32.3%	33.8%	35.8%	35.9%
<b>R8</b>	44.8%	50.3%	50.7%	51.2%	49.9%	53.0%	57.0%	63.0%	58.3%
<b>R9</b>	96.0%	95.1%	94.0%	92.4%	92.1%	89.8%	90.3%	89.7%	92.6%
<b>R10</b>	260.5%	133.2%	131.4%	128.7%	130.2%	125.3%	118.4%	111.2%	104.8%

Class 2: non life insurer with big size

Variable	1999	2000	2001	2002	2003	2004	2005	2006	2007
<b>R1</b>	-4.3%	1.0%	0.7%	-3.1%	3.3%	1.3%	1.7%	2.9%	1.9%
<b>R2</b>	3.9%	3.3%	2.4%	2.7%	5.6%	4.6%	6.2%	6.8%	5.4%
<b>R3</b>	10.8%	4.2%	-0.7%	2.4%	-13.3%	22.9%	21.0%	20.1%	15.1%
<b>R5</b>	2.0%	4.2%	3.9%	3.4%	7.4%	8.2%	9.9%	10.2%	8.6%
<b>R4</b>	61.1%	60.5%	60.6%	60.9%	61.3%	64.4%	65.3%	66.2%	65.6%
<b>R5</b>	29.8%	28.0%	25.9%	24.9%	25.5%	25.5%	26.9%	28.3%	29.2%
<b>R6</b>	132.4%	127.9%	123.4%	121.9%	123.1%	122.7%	124.4%	127.8%	128.7%
<b>R7</b>	134.9%	127.5%	127.7%	127.5%	124.1%	128.5%	133.8%	134.6%	138.1%
<b>R8</b>	189.5%	203.8%	191.5%	188.2%	196.2%	210.5%	223.6%	234.3%	240.0%
<b>R9</b>	102.5%	97.5%	100.2%	94.4%	91.4%	89.7%	90.0%	88.4%	90.8%
<b>R10</b>	135.7%	75.4%	76.1%	76.9%	76.1%	73.1%	68.9%	68.1%	66.7%

Class 3: insurers with all kind of line of business

Variable	1999	2000	2001	2002	2003	2004	2005	2006	2007
<b>R1</b>	12.3%	5.6%	5.1%	3.0%	5.6%	5.0%	4.9%	5.4%	5.5%
<b>R2</b>	1.1%	1.1%	1.1%	1.3%	2.3%	3.2%	3.4%	3.9%	3.8%
<b>R3</b>	24.8%	6.7%	5.9%	5.5%	10.9%	15.2%	15.9%	17.5%	16.7%
<b>R5</b>	1.4%	2.3%	3.9%	6.3%	7.3%	10.0%	10.7%	11.8%	13.7%
<b>R4</b>	78.4%	77.8%	79.5%	79.8%	81.2%	82.8%	83.2%	84.0%	84.0%
<b>R5</b>	18.7%	19.1%	19.0%	18.8%	18.9%	19.6%	20.2%	20.9%	21.2%
<b>R6</b>	125.0%	128.9%	128.0%	125.0%	127.1%	129.9%	134.4%	137.0%	135.9%
<b>R7</b>	342.2%	306.8%	340.4%	326.4%	435.2%	427.5%	398.2%	398.3%	405.3%
<b>R8</b>	527.7%	497.1%	496.1%	481.2%	516.1%	532.0%	548.4%	516.7%	526.5%
<b>R9</b>	97.6%	89.1%	91.1%	91.8%	106.1%	104.3%	97.9%	99.6%	103.9%
<b>R10</b>	80.0%	46.4%	41.6%	41.6%	37.4%	36.3%	36.9%	36.0%	34.8%

Class 4: life insurers with big size

Variable	1999	2000	2001	2002	2003	2004	2005	2006	2007
<b>R1</b>	10.2%	6.1%	4.8%	4.7%	4.4%	4.1%	3.9%	3.8%	3.8%
<b>R2</b>	1.8%	1.0%	1.1%	0.5%	1.3%	1.6%	1.7%	1.9%	1.8%
<b>R3</b>	17.7%	10.0%	14.0%	6.9%	15.2%	14.8%	15.8%	16.7%	13.7%
<b>R5</b>	-2.8%	1.5%	3.9%	-0.8%	6.3%	7.6%	9.0%	9.7%	8.0%
<b>R4</b>	78.1%	76.4%	77.7%	80.6%	82.0%	81.4%	81.9%	80.7%	78.7%
<b>R5</b>	12.7%	10.5%	9.2%	9.9%	9.5%	8.8%	8.8%	9.6%	9.9%
<b>R6</b>	147.3%	137.3%	119.9%	122.4%	116.5%	113.2%	112.0%	113.4%	113.2%
<b>R7</b>	567.7%	466.2%	470.8%	508.2%	687.2%	576.6%	679.4%	668.1%	679.4%
<b>R8</b>	1190.6%	821.6%	732.2%	715.6%	826.5%	762.4%	780.7%	604.3%	523.0%
<b>R9</b>	79.1%	69.5%	80.0%	93.7%	110.9%	88.0%	100.1%	128.9%	157.4%
<b>R10</b>	49.7%	31.8%	27.7%	25.3%	23.0%	24.2%	20.4%	19.7%	21.4%

Class 5: life insurers with small size

Variable	1999	2000	2001	2002	2003	2004	2005	2006	2007
<b>R1</b>	13.5%	7.0%	6.1%	5.9%	6.8%	5.1%	4.9%	6.0%	5.6%
<b>R2</b>	1.7%	2.1%	0.1%	-1.2%	0.9%	0.5%	1.8%	1.3%	6.2%
<b>R3</b>	8.7%	5.2%	-16.2%	5.0%	-32.6%	1.9%	16.3%	7.3%	16.0%
<b>R5</b>	11.2%	45.3%	-19.4%	-23.6%	18.0%	-23.7%	32.1%	18.6%	25.8%
<b>R4</b>	93.7%	90.4%	92.6%	92.4%	92.2%	92.7%	90.8%	93.5%	93.8%
<b>R5</b>	13.9%	15.0%	14.6%	13.3%	13.3%	13.2%	13.4%	13.8%	16.2%
<b>R6</b>	138.5%	141.4%	142.0%	127.3%	125.8%	125.7%	126.3%	127.1%	136.4%
<b>R7</b>	1209.6%	1220.6%	1253.1%	1202.2%	1189.5%	1303.8%	1272.1%	1292.6%	1353.3%
<b>R8</b>	1260.1%	1255.8%	1356.5%	1181.8%	1269.8%	1753.7%	1444.6%	1437.6%	1465.8%
<b>R9</b>	108.0%	108.7%	112.6%	135.3%	110.0%	100.5%	112.6%	112.4%	116.8%
<b>R10</b>	19.2%	9.5%	8.7%	8.8%	9.7%	8.7%	8.5%	8.5%	8.1%

*Source:* own elaboration

## Annex II: Estimation of the Multinomial state space model

Class 2: non life insurer with big size

Variable	1999	2000	2001	2002	2003	2004	2005	2006	2007
intercept	<b>6.67</b>	<b>6.97</b>	<b>7.27</b>	<b>7.57</b>	<b>7.88</b>	<b>8.18</b>	<b>8.48</b>	<b>8.78</b>	<b>9.08</b>
R1	-1.60	-1.51	-1.43	-1.34	-1.26	-1.17	-1.09	-1.00	-0.92
R2	-1.09	-1.10	-1.12	-1.13	-1.14	-1.16	-1.17	-1.19	-1.20
R3	2.05	2.34	2.62	<b>2.90</b>	<b>3.19</b>	<b>3.47</b>	<b>3.76</b>	<b>4.04</b>	<b>4.32</b>
R4	<b>-2.60</b>	<b>-2.40</b>	<b>-2.21</b>	<b>-2.01</b>	<b>-1.82</b>	<b>-1.62</b>	<b>-1.43</b>	-1.23	-1.04
R5	<b>-8.66</b>	<b>-8.64</b>	<b>-8.62</b>	<b>-8.61</b>	<b>-8.59</b>	<b>-8.58</b>	<b>-8.56</b>	<b>-8.54</b>	<b>-8.53</b>
R6	<b>-8.39</b>	<b>-8.12</b>	<b>-7.85</b>	<b>-7.58</b>	<b>-7.31</b>	<b>-7.04</b>	<b>-6.77</b>	<b>-6.51</b>	<b>-6.24</b>
R7	-3.70	-3.26	-2.82	-2.38	-1.94	-1.50	-1.06	-0.62	-0.18
R8	1.18	2.16	3.14	<b>4.12</b>	<b>5.10</b>	<b>6.08</b>	<b>7.06</b>	<b>8.04</b>	<b>9.02</b>
R9	-1.27	-1.77	<b>-2.27</b>	<b>-2.77</b>	<b>-3.27</b>	<b>-3.77</b>	<b>-4.27</b>	<b>-4.77</b>	<b>-5.27</b>
R10	<b>-6.38</b>	<b>-6.64</b>	<b>-6.91</b>	<b>-7.18</b>	<b>-7.44</b>	<b>-7.71</b>	<b>-7.98</b>	<b>-8.24</b>	<b>-8.51</b>

Class 3: insurers with all kind of line of business

Variable	1999	2000	2001	2002	2003	2004	2005	2006	2007
intercept	0.63	0.93	<b>1.23</b>	<b>1.53</b>	<b>1.83</b>	<b>2.14</b>	<b>2.44</b>	<b>2.74</b>	<b>3.04</b>
R1	<b>3.15</b>	<b>3.23</b>	<b>3.32</b>	<b>3.40</b>	<b>3.49</b>	<b>3.57</b>	<b>3.65</b>	<b>3.74</b>	3.82
R2	<b>-4.10</b>	<b>-4.11</b>	<b>-4.13</b>	<b>-4.14</b>	<b>-4.16</b>	<b>-4.17</b>	<b>-4.19</b>	<b>-4.20</b>	<b>-4.21</b>
R3	1.26	1.55	1.83	2.12	2.40	2.68	2.97	3.25	3.54
R4	<b>2.41</b>	<b>2.61</b>	<b>2.80</b>	<b>3.00</b>	<b>3.19</b>	<b>3.39</b>	<b>3.58</b>	<b>3.78</b>	<b>3.98</b>
R5	<b>-10.16</b>	<b>-10.14</b>	<b>-10.13</b>	<b>-10.11</b>	<b>-10.09</b>	<b>-10.08</b>	<b>-10.06</b>	<b>-10.05</b>	<b>-10.03</b>
R6	-1.20	-0.93	-0.66	-0.40	-0.13	0.14	0.41	0.68	0.95
R7	<b>4.04</b>	<b>4.48</b>	<b>4.92</b>	<b>5.36</b>	<b>5.80</b>	<b>6.24</b>	<b>6.68</b>	<b>7.12</b>	<b>7.56</b>
R8	<b>8.30</b>	<b>9.27</b>	<b>10.25</b>	<b>11.23</b>	<b>12.21</b>	<b>13.19</b>	<b>14.17</b>	<b>15.15</b>	<b>16.13</b>
R9	1.00	0.50	<b>0.00</b>	<b>-0.50</b>	<b>-1.00</b>	<b>-1.50</b>	<b>-2.00</b>	<b>-2.50</b>	<b>-3.00</b>
R10	<b>-6.05</b>	<b>-6.32</b>	<b>-6.58</b>	<b>-6.85</b>	<b>-7.11</b>	<b>-7.38</b>	<b>-7.65</b>	<b>-7.91</b>	<b>-8.18</b>

Class 4: life insurers with big size

Variable	1999	2000	2001	2002	2003	2004	2005	2006	2007
intercept	<b>2.59</b>	<b>2.89</b>	<b>3.19</b>	<b>3.50</b>	<b>3.80</b>	<b>4.10</b>	<b>4.40</b>	<b>4.70</b>	<b>5.01</b>
R1	-0.85	-0.77	-0.68	-0.60	-0.51	-0.43	-0.35	-0.26	-0.18
R2	-2.85	-2.86	-2.88	-2.89	-2.91	-2.92	-2.94	-2.95	-2.96
R3	1.00	1.29	1.57	1.85	2.14	2.42	2.71	2.99	3.27
R4	0.06	0.25	0.45	0.64	0.84	1.03	1.23	1.42	1.62
R5	<b>-15.01</b>	<b>-15.00</b>	<b>-14.98</b>	<b>-14.96</b>	<b>-14.95</b>	<b>-14.93</b>	<b>-14.92</b>	<b>-14.90</b>	<b>-14.89</b>
R6	3.29	<b>3.56</b>	<b>3.83</b>	<b>4.10</b>	<b>4.37</b>	<b>4.64</b>	<b>4.91</b>	<b>5.18</b>	<b>5.45</b>
R7	<b>6.26</b>	<b>6.70</b>	<b>7.14</b>	<b>7.58</b>	<b>8.02</b>	<b>8.46</b>	<b>8.90</b>	<b>9.34</b>	<b>9.78</b>
R8	<b>8.68</b>	<b>9.66</b>	<b>10.64</b>	<b>11.62</b>	<b>12.60</b>	<b>13.58</b>	<b>14.56</b>	<b>15.54</b>	<b>16.52</b>
R9	-1.08	-1.58	<b>-2.08</b>	<b>-2.58</b>	<b>-3.08</b>	<b>-3.58</b>	<b>-4.08</b>	<b>-4.58</b>	<b>-5.08</b>
R10	<b>-12.08</b>	<b>-12.35</b>	<b>-12.62</b>	<b>-12.88</b>	<b>-13.15</b>	<b>-13.42</b>	<b>-13.68</b>	<b>-13.95</b>	<b>-14.22</b>

Class 5: life insurers with small size

Variable	1999	2000	2001	2002	2003	2004	2005	2006	2007
intercept	<b>-4.85</b>	<b>-4.55</b>	<b>-4.25</b>	<b>-3.95</b>	<b>-3.64</b>	<b>-3.34</b>	<b>-3.04</b>	<b>-2.74</b>	<b>-2.43</b>
R1	0.49	0.57	0.66	0.74	0.83	0.91	1.00	1.08	1.17
R2	-1.15	-1.17	-1.18	-1.20	1.21	-1.23	-1.24	-1.25	-1.27
R3	-0.12	0.16	0.45	0.73	1.01	1.30	1.58	1.87	2.15
R4	<b>5.12</b>	<b>5.31</b>	<b>5.51</b>	<b>5.70</b>	<b>5.90</b>	<b>6.09</b>	<b>6.29</b>	<b>6.48</b>	<b>6.68</b>
R5	<b>-9.07</b>	<b>-9.05</b>	<b>-9.03</b>	<b>-9.02</b>	<b>-9.00</b>	<b>-8.99</b>	<b>-8.97</b>	<b>-8.95</b>	<b>-8.94</b>
R6	-1.54	-1.27	-1.00	-0.73	-0.46	-0.20	0.07	0.34	0.61
R7	<b>10.32</b>	<b>10.76</b>	<b>11.20</b>	<b>11.64</b>	<b>12.08</b>	<b>12.52</b>	<b>12.96</b>	<b>13.40</b>	<b>13.84</b>
R8	<b>11.06</b>	<b>12.04</b>	<b>13.02</b>	<b>14.00</b>	<b>14.97</b>	<b>15.95</b>	<b>16.93</b>	<b>17.91</b>	<b>18.89</b>
R9	-0.88	-1.38	-1.88	-2.38	<b>-2.88</b>	<b>-3.38</b>	<b>-3.88</b>	<b>-4.38</b>	<b>-4.88</b>
R10	<b>-12.41</b>	<b>-12.68</b>	<b>-12.95</b>	<b>-13.21</b>	<b>-13.48</b>	<b>-13.75</b>	<b>-14.01</b>	<b>-14.28</b>	<b>-14.54</b>

Note: Bold numbers reflect posterior means with HDP (Highest Posterior Density) not including zero values at 0.95.

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